

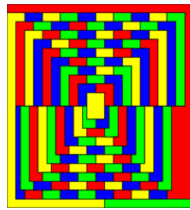
# Student Research Talks (StReeTs)

Mason Experimental Geometry Lab (MEGL)

## Coloring Squares of Planar Graphs with no 4-cycles and no 5-cycles

**Bobby Jaeger**

Department of Mathematics Sciences  
George Mason University



### Abstract

The famous Four Color Theorem states that any planar graph can be properly colored using at most four colors. However, if we want to properly color the square of a planar graph (or alternately, color the graph using distinct colors on vertices at distance up to two from each other), we will always require at least  $\Delta + 1$  colors, where  $\Delta$  is the maximum degree in the graph. For all  $\Delta$ , Wegner constructed planar graphs (even without 3-cycles) that require about  $\frac{3}{2}\Delta$  colors for such a coloring.

To prove a stronger upper bound, we consider only planar graphs that contain no 4-cycles and no 5-cycles (but which may contain 3-cycles). Zhu, Lu, Wang, and Chen showed that for a graph  $G$  in this class with  $\Delta \geq 9$ , we can color  $G^2$  using no more than  $\Delta + 5$  colors. We improve this result, showing that for the same class of graphs, as long as  $\Delta$  is sufficiently high, at most  $\Delta + 3$  colors are needed. Our approach uses the discharging method, and the result extends to list-coloring and other related coloring concepts as well. This is joint work with Dan Cranston.

Date: Friday, October 27, 2017

Time: 2:30pm–3:30pm

Place: Exploratory Hall 4106

**Pizza and soda will be served at the presentation.**

For further information or for special accommodations, please contact Sean Lawton via email at [seanlawton@gmail.com](mailto:seanlawton@gmail.com) or drop by the MEGL.