

Student Research Talks (StReeTs)

Mason Experimental Geometry Lab (MEGL)

The Perfect Closure R^∞

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Abstract

In rings of characteristic $p > 0$ we have the Frobenius map $f : R \rightarrow R$ where $f(r) = r^p$, which is a homomorphism since in this context $(x + y)^p = x^p + y^p$. If a ring R is reduced, meaning it has no elements x such that $x^n = 0$ for some n , then this map is injective. However, it is very far from being surjective in general (e.g. for $R = \mathbb{F}_p[x]$, no element maps to the variable x under f).

Let (R, m, k) be a reduced local Noetherian ring of characteristic $p > 0$. In order to make the Frobenius map surjective we can extend R to its *perfect closure* R^∞ which contains all $1/p^n$ -th roots of elements of R . In most cases this extension is no longer a Noetherian ring (e.g. for $R = \mathbb{F}_p[x]$, in R^∞ the chain $(x) \subset (x^{\frac{1}{p}}) \subset (x^{\frac{1}{p^2}}) \subset \dots$ never stabilizes), thus fundamental concepts such as associated primes and depth require more general definitions. In this talk we will discuss how the structures of R and R^∞ affect one another with respect to these ring theoretic concepts. Many links are known, but many questions remain which ultimately may help determine features about the structure of a given ring R .

Date: Friday, November 20, 2015

Time: 2:30pm–3:30pm

Place: Exploratory Hall 4106

Pizza and soda will be served at the presentation.

For further information or for special accommodations, please contact Sean Lawton via email at seanlawton@gmail.com or drop by the MEGL.