

# STudent REsearch TalkS (StReTs)

Mason Experimental Geometry Lab (MEGL)

## Matrix Algebras: Equivalent Ring Relations and Special Presentations

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### Abstract

A classic result in noncommutative ring theory states that a ring  $R$  is an  $n \times n$  matrix ring if, and only if,  $R$  contains  $n^2$  matrix units  $\{e_{ij}\}_{1 \leq i, j \leq n}$ , in which case  $R \cong M_n(S)$  where  $S$  is a subring of  $R$  that can be described completely in terms of the matrix units. A lesser known result states that a ring  $R$  is an  $(m+n) \times (m+n)$  matrix ring, so  $R \cong M_{m+n}(S)$  for some ring  $S$ , if, and only if,  $R$  contains three elements  $a$ ,  $b$ , and  $f$  satisfying the two relations  $af^m + f^nb = 1$  and  $f^{m+n} = 0$ . In this talk, we investigate algebras over a commutative ring (or field) with elements  $c$  and  $f$  satisfying the two relations  $c^i f^m + f^n c^j = 1$  and  $f^{m+n} = 0$ . Surprisingly little is known here about the structure of these algebras and about the underlying ring  $S$  for most cases of the integers  $i$ ,  $j$ ,  $m$ , and  $n$ . Questions whether  $S$  is non-trivial or not turn out to be surprisingly difficult to answer, let alone describing the structure of these algebras or of  $S$  in general.

Date: Friday, February 6, 2015

Time: 2:30pm–3:30pm

Place: Exploratory 4106, Fourth Floor

**Pizza and soda will be served at the presentation.**

For further information or for special accommodations, please contact Sean Lawton via email at [seanlawton@gmail.com](mailto:seanlawton@gmail.com) or drop by the MEGL.