Asymptotic Dimension

View point:

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View point: Look at the global dimension of a space. The classical view of dimension is focused on the "degree's" of freedom, which is a local property.

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Definition (Asymptotic Dimension)

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2. A collection of families, $\mathcal{U}^i = \{U^i_\alpha\}_{\alpha \in A}$ is λ *disjoint* if $dist(U^i_\alpha, U^i_\beta) > \lambda$ for $\alpha \neq \beta$

Definition

The *asymptotic dimension with linear control* $\ell\text{-asdim}_* X$ of a metric space X is defined as follows: $\ell\text{-asdim}_* X \leq n$ if there is $c > 0$ such that for every $R < \infty$ there is $\lambda > R$ such that $(\lambda, c\lambda)$-dim$X \leq n$.

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3. $\ell\text{-asdim}_* X = 0$, for any compact metric space.

Assouad Nagata Dimension

Definition

The *asymptotic Assouad-Nagata dimension* $\mathrm{AN\text{-}asdim}\, X \leq n$ if there is $c > 0$ such that $(\lambda, c\lambda)$-dim$X \leq n$ for $\lambda > r_0$ for some r_0.